Joule heating of plasma in current-carrying coronal magnetic loops

V.V. Zaitsev[1], **A.A. Stepanov**[2]

[1]Institute of Applied Physics, Nizhny Novgorod, Russia
[2]Central Astronomical Observatory, Pulkovo, St.Peterburg, Russia

Introduction

Already first X-ray observations made on "Skylab" in 1969 (Vaiana, et al., 1970) have shown that solar corona consists of a lots of magnetic field loops, filled with a hot plasma. The average temperature of plasma in these loops is $(1.3 \div 1.7) \cdot 10^6$ K, whereas plasma density inside of them at least $2-4$ times exceeds the average coronal plasma density (Golub, et al., 1974). Observations of X-ray loops indicate that their heating happens on the time intervals about the time of radiative energy losses. More rapid temperature increase in the loop however is also possible. Nolte, et al. (1979) suppose that the observed behavior of the temperature dynamics can be explained by the presence of two heating components: slowly changing (or constant) component, and an impulsive one, causing rapid increase of the brightness of X-ray loops. Rosner et al., (1978) assumed the presence inside a coronal magnetic loop of a heating source of indefinite nature and considered a steady-state heating of such a magnetic loop. In this work a steady-state heating of bases of current-carrying magnetic loops is considered. As a source of heating we assume the Joule energy dissipation of currents, which are generated by a convective flow of partially ionized photospheric plasma in the loop's foot-points (Zaitsev and Khodachenko, 1997, Zaitsev et al., 1998). Below we'll show, firstly, – that coronal magnetic loops with currents should be hot (this follows from the equilibrium conditions), and, secondary, – we'll find a relation between a maximum temperature in bases of a loop and a velocity of photospheric convection.

Equilibrium of a magnetic current-carrying loop

Before to analyze an energy dissipation of currents, running in a magnetic loop formed by a converging convective flow of a photospheric plasma in joining points of few supergranulas, let us consider a balance of kinetic and magnetic pressures in bases of the loop. From the condition of balance of vertical magnetic tube in a dynamo region, where a radial component of a plasma convection velocity $V_r \neq 0$, and a current density $\mathbf{j}$ is perpendicular to a magnetic field $\mathbf{B}$

$$\frac{\partial P}{\partial r} = \frac{1}{c}(j_\varphi B_z - j_z B_\varphi) = -\frac{\sigma V_r}{c^2}\frac{B^2}{1+\alpha B^2} \tag{1}$$

where $\sigma = \dfrac{e^2 n}{m_e(\nu_{ei}+\nu_{ea})}$ – Coulomb conductivity, $\alpha = \dfrac{\sigma F^2}{(2-F)c^2 n m_i \nu_{ia}}$, n – density of electrons, F – relative density of neutrals, ν_{ei} and ν_{ea} are respectively effective frequencies of electron-ion and electron-atom collisions, it follows that kinetic pressure in a magnetic tube increases towards its periphery. On a boundary of such a magnetic tube $(r = r_0)$

the pressure is of the order of magnitude of a magnetic pressure in the central part of the tube:

$$P(r_0) - P(0) = \frac{B_z^2(0)}{8\pi(1 - b^2 ln \mid (1+b^2)/b^2 \mid)}, \tag{2}$$

where $b^2 = \left(\frac{B_\varphi(r_0)}{B_z(r_0)}\right)^2$, $B_z(0)$ is a vertical component of a magnetic field on the axis of the tube. If to take that convective motion of plasma takes place up to heights $h = 500$ km above the photospheric level ($\tau_{5000} = 1$), then the standard atmosphere pressure on these heights appears to be about $1.35 \cdot 10^3$ dyne $\cdot$ cm^{-2} (Vernazza, et al., 1981). Therefore, a magnetic tube with the standard atmosphere pressure is in equilibrium with a convective flow only if the value of a magnetic field in the tube is less then approximately 10^2 G. For more intensive magnetic tubes $B_z(0) = 10^3$ G and the twisting ratio $b^2 = 1$ the balance with a convective flow could be reached only in the case when gas pressure in the tube is $P(r_0) \approx 1.3 \cdot 10^5$ dyne $\cdot$ cm^{-2}. For usual photospheric temperatures such values of pressure are realized only in regions, situated about 25 km above the photosphere and deeper ($h < -25$ km). Thus, for the balance to take place in a magnetic tube plasma in the tube should be hot. As far as the existence in solar corona of intensive magnetic loops with the fields of a few kilogauss is of no doubt, then it seems that photospheric bases of such loops should be filled with a hot plasma (if one assumes that generation of these loops is caused by converging flows of photospheric plasma).

Joule dissipation of currents energy

What are the mechanisms, which cause the heating of magnetic loops in the photospheric dynamo region? One of the possible mechanisms can be Joule heating, taking place during energy dissipation of currents, generated in the magnetic tube by a convection flow. As it was already noted, the current density $\mathbf{j}$ is perpendicular to the self-consistent magnetic field $\mathbf{B}$ in this case, and the Joule heating rate is determined by Cowling conductivity. This appears to be quite an effective heating mechanism. The steady-state Joule energy release in the conditions of a sunspot with an Evershed's flow was studied first by Sen and White (1972) in the approximation of a purely vertical magnetic field ($B_\varphi = 0$). For a magnetic tube with a convective plasma flow a value of the Joule energy release ratio depends on both components of a magnetic field ($B_\varphi \neq 0, \quad B_z \neq 0$). The energy, $q_J = (\mathbf{E} + \frac{1}{c}\mathbf{V} \times \mathbf{B})\mathbf{j}$, released in 1 s in a unit volume in the case of $\mathbf{E} = \{E_r, 0, 0\}$, $\mathbf{V} = \{V_r, 0, V_z\}$ and $\mathbf{j} = \{0, j_\varphi, j_z\}$ is

$$q_J = \frac{j^2}{\sigma_C}, \quad j^2 = j_z^2 + j_\varphi^2 = \frac{\sigma^2 V_r^2}{c^2}\frac{B^2}{(1+\alpha B^2)^2}, \quad \sigma_C = \frac{\sigma}{1+\alpha B^2}, \tag{3}$$

or

$$q_J = \frac{\sigma V_r^2 B^2}{(1+\alpha B^2)c^2}. \tag{4}$$

It is interesting to note here that the equation (1) can be considered now in a sense of a heat-transfer equation, which describes a pressure increase (due to the Joule heating) in direction from the tube's center towards its periphery:

$$-V_r \frac{\partial P}{\partial r} = q_J = \frac{\sigma V_r^2 B^2}{(1+\alpha B^2)c^2}. \tag{5}$$

The expression for the rate of Joule energy release becomes to be the most simple in a case of intensive magnetic tubes, when $\alpha B^2 \gg 1$

$$q_J \approx \frac{\sigma V_r^2}{\alpha c^2} = \frac{n m_i \nu_{ia} V_r^2(r)(2-F)}{F^2}. \tag{6}$$

As it follows from (6), if V_r increases with the r increase, then the energy release also increases in this direction. In this case one can expect that the surface of the magnetic tube will be hotter than its inner regions.

Maximum temperature

Let's consider now the temperature evolution with height in a photospheric part of a magnetic tube. By this, we'll suppose that plasma pressure is constant and is determined by a value of a magnetic field in the tube. In this case the temperature evolution is determined by a balance between the Joule heating and energy losses due to the thermo-conductivity, as well as optical radiation of plasma. Because of the rapid temperature increase plasma in the tube can be assumed to be completely ionized, and the following state equation $P = 2nk_BT$ can be used. Here P, n and T are the averaged over the tube's cross-section values of pressure, density and temperature. For the case of intensive magnetic loops ($\alpha B^2 \gg 1$) the average (over the tube cross-section) Joule energy release rate can be written as $q_J = \xi n^2$ erg $\cdot$ cm$^{-3}\cdot$ s^{-1}, where $\xi = 8.3 \cdot 10^{-33} V_0^2$. The radiative energy losses function in the temperature interval $5 \le logT \le 7$ we approximate as the following (Golub and Pasachoff, 1998): $q_J = \chi n^2 T^{-1}$erg $\cdot$ cm$^{-3}\cdot$ s^{-1}, where $\chi = 10^{-16.22}$. The thermo-conductivity heat flux along a magnetic field is $F_c = -k_0 T^{5/2}(z)\dfrac{dT}{dz}$, where $k_0 = 9.2 \cdot 10^{-7}$ cgs units. For simplicity's sake assume a small twisting of the tube ($B_\varphi^2 \ll B_z^2$). In this case the magnetic field of the tube is approximately parallel to the $z-$ axis.

Within the frame of the assumptions made above let's consider the thermo-equilibrium equation

$$\frac{d}{dz} k_0 T^{5/2} \frac{dT}{dz} = \frac{P^2 \chi}{4k_B^2 T^3} - \frac{P^2 \xi}{4k_B^2 T^2} \tag{7}$$

We take the $z = 0$ level on some height under the photosphere where magnetic pressure of the tube begins to exceed a value of an unperturbed atmosphere's gas pressure, and the tube starts to be heated. On this zero-level there should be determined the values of temperature and its derivative:

$$T(z=0) = T_0, \qquad \left.\frac{dT}{dz}\right|_{z=0} = \left(\frac{dT}{dz}\right)_0 . \tag{8}$$

Let's assume that the temperature increases with height upward from the point $z = 0$ and reaches its maximum value on the level $z = z_1$, i.e.

$$T(z = z_1) = T_1, \qquad \frac{dT}{dz}|_{z=z_1} = 0\ . \tag{9}$$

After multiplying (7) on $T^{5/2}\frac{dT}{dz}$ and integration over the coordinate, starting from z_1 one obtains

$$\frac{1}{2}k_0 T^5 \left(\frac{dT}{dz}\right)^2 = \frac{P^2}{4k_B^2}[2\chi(\sqrt{T} - \sqrt{T_1}) - \frac{2\xi}{3}(T^{3/2} - T_1^{3/2})]. \tag{10}$$

As far as $T_1 \gg T_0$, and the heat flux $k_0 T_0^{5/2}(dT/dz)_{z=z_0}$ is small in comparison with $2\chi P^2 T_1^{1/2}/(4k_B^2)$ and $2\xi P^2 T_1^{3/2}/(12k_B^2)$, then from (10) one obtains an equation for the maximal temperature $-2\chi T_1^{1/2} + \frac{2}{3}\xi T_1^{3/2} = 0$, which gives a value of T_1:

$$T_1 = \frac{3\chi}{\xi} \approx \frac{2\cdot 10^{16}}{V_0^2}. \tag{11}$$

For the values of a horizontal component of a convection velocity $V_0 = 0.3 \div 1$ km $\cdot$ s^{-1} (11) gives $T_1 \approx 2\cdot(10^6 \div 10^7)$ K.

Let's determine the height on which the temperature reaches the obtained maximal values. After exclusion of ξ from (10) with taking account of (11) and taking a square root the equation (10) transforms to

$$T^{9/4}\frac{dT}{dz} = \left(\frac{P^2\chi}{k_B^2 k_0}\right)^{1/2}\left(1 - \frac{T}{T_1}\right)^{1/2}. \tag{12}$$

Integration of (12) over z from 0 till z_1, for $T_0/T_1 \ll 1$ yields the relation between z_1 (the height of the maximal temperature of the photospheric bases of a magnetic tube), pressure P and a value of the maximal temperature T_1:

$$z_1 P = \left(\frac{k_B^2 k_0}{\chi}\right)^{1/2} I T^{13/4}, \tag{13}$$

where $I = \int_0^1 t^{9/4}(1-t)^{-1/2}dt$ is dimensionless value of the order of a unit. If one will assume that the pressure in a photospheric base of a magnetic loop is determined by a pressure of a magnetic field, i.e. $P = \frac{B_z^2}{16\pi(1 - b^2 ln \mid (1+b^2)/b^2 \mid)}$ (see (2)), and take formula (11) into account, then from (13) one will obtain $z_1 \approx 0.8\cdot 10^5 B_3^{-2} v_0^{-13/2}$ cm, where $B_3 = B_z(0)/10^3$[G], and $v_0 = V_0/10^5$[cm $\cdot$ s^{-1}] are dimensionless values of a magnetic field and horizontal convection velocity. For $B_z(0) = 10^3$ G and $V_0 \approx 0.3$ km $\cdot$ s^{-1} one can obtain $z_1 \approx 10^3$ km. Thus the temperature reaches a maximal value on a scale of the order of the scale of a dynamo region in the loop's foot-point. This scale decreases, i.e. the temperature increases faster, if the convection velocity and magnetic field increase. And vice versa – for small velocities (for example $V_0 \approx 0.1$ km $\cdot$ s^{-1}) the maximal temperature is not reached at all, since the scale z_1 becomes to be greater then the dynamo region's scale in the photospheric bases of the loop.

There is no convection in a chromospheric part of a magnetic loop ($V_r = 0$) and Joule heating is uneffective here, since the plasma conductivity increases significantly in comparison with the Cowling conductivity of photospheric partially ionized plasma. Therefore the temperature of plasma in the bases of a magnetic loop decreases rather quickly with height increase. This happens because of the energy losses due to the thermo-conductivity and radiation. A characteristic scale on which the temperature decreases significantly is also of the order of magnitude of the scale z_1, i.e. about 10^3 km. If however an additional heating source there exist in corona (this heating source could be caused for example by dissipation of Alfven or acoustic waves, Joule dissipation of coronal currents) then the plasma temperature in the tube can continue to grow with height, reaching its maximal value in the top of the loop. In this case for sufficiently low magnetic loops with a height being less then the gravitational height scale $H = 5 \cdot 10^3 T$ cm the following relation between a maximal temperature in the top of a loop T_{max}, its length l and plasma pressure P inside is fulfilled (Rosner et al., 1978; Serio et al., 1981):

$$T_{max} \approx 1.4 \cdot 10^3 (Pl)^{1/3} \text{ K} \tag{14}$$

The heating source q_H of an uncertain nature should satisfy the following relation (Rosner et al., 1978):

$$q_H \approx 9.8 \cdot 10^4 P^{7/6} l^{-5/6} \text{ erg} \cdot \text{cm}^{-3} \cdot \text{ s}^{-1} \tag{15}$$

which means that the pressure, influencing the radiative energy losses ratio, has a positive correlation with the heating rate.

Conclusion

We have shown above that current-carrying magnetic loops with a potential magnetic field of about 10^3 G and convective flows of photospheric plasma should have hot bases. The source of their heating is connected with a Joule dissipation of energy of current in the photospheric foot-points. The high rate of the energy release is caused by Cowling conductivity. In particular, for horizontal component of a velocity of convection flow $0.3 \div 1$ km $\cdot$ s^{-1} the maximal temperature in the bases of the loop can reach values of $2 \cdot (10^6 \div 10^7)$ K. Such loops can be associated with bright X-ray points, which are observed by "Skylab" and other space observatories.

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